Terminal Sliding Mode Control for MIMO T-S Fuzzy Systems

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***Abstract*— In this paper, we present a terminal sliding mode control for MIMO T-S fuzzy systems. It is shown that the concept of extreme matrices is used to determine the upper bound information of the interactions of the fuzzy subsystems first, and a terminal sliding mode controller and a set of sliding variables are then designed to enable finite time reachability of the system origin. Simulation results are presented in support of the proposed approach.**

***Keywords*—terminal sliding mode control, T-S fuzzy systems, extreme matrices, fuzzy control**

I. Introduction

Fuzzy logic control has been proven to be a successful control approach for a class of complex nonlinear systems, see [1-3], for example. In the conventional fuzzy logic control design, T-S fuzzy model is used to model the complex nonlinear system first, so that a local controller can be designed for each local fuzzy subsystem. The global fuzzy logic control law can then be obtained by aggregating all the local controllers using the fuzzy inference law.

Recently, many new stability analyses and controller design results for T-S fuzzy systems have appeared in the fuzzy control literature [4-6]. The system states can be proven to be asymptotically stable, that is, the convergence in such systems can at best be exponentially with infinite settling time. It is therefore interesting to design controller for T-S fuzzy systems with finite time convergence property. Comparing to the infinite time controller, due to the non-Lipschitz property, the finite time controller is able to not only provide faster convergence rate, but also perform better in the presence of system uncertainties and disturbances [7, 8].

Terminal sliding mode (TSM) control was developed in [9-11] to achieve finite time convergence of the system dynamics on the sliding mode surface. In [9, 11], the hierarchical terminal sliding mode structure was proposed for SISO control systems such that the sliding variables can converge to zero sequentially and then, the system states can reach the system origin in finite time. A new TSM surface was proposed in [10] for MIMO linear systems. It was shown that by suitably designing the parameter matrices of the sliding variable, on the sliding mode surface, the system states converge to zero in finite time.

In this paper, we present a systematic TSM control design for MIMO T-S fuzzy systems. We first introduce the concept of the extreme matrices [12, 13] where, in each subspace, a dominant membership function, together with its local subsystem, dominates the global T-S fuzzy system. A set of extreme matrices describing the upper bounds of all the interactions among the local fuzzy subsystems in the worst stability case are then derived. A TSM control law and a set of TSM surfaces using these extreme matrices as the bounded information are designed to guarantee finite time convergence of the closed-loop system. It is the extreme matrices that simplify the TSM control design for MIMO T-S fuzzy systems.

To our best knowledge, there is little work dealing with finite time control for T-S fuzzy systems in the literature at present stage. The proposed control scheme expands the class of nonlinear systems that can be handled using TSM technique.

The rest of the paper is organized as follows. Section II introduces the MIMO T-S fuzzy systems. Section III discusses the concept about the extreme matrices. Section IV presents TSM control design for MIMO T-S fuzzy systems. Section V presents a numerical simulation example in support of the developed control scheme. Section VI gives conclusions.

II. Problem Formulation

Consider the MIMO T-S Fuzzy system with both fuzzy inference rules and local analytic linear models as follows:

$$\begin{aligned} R^i:\ & IF \quad x_1 \ is \ F_1^i \ AND \ldots x_n \ is \ F_n^i \\ & THEN \quad \dot{\boldsymbol{x}}_1 = \boldsymbol{A}_{11}{}^i \boldsymbol{x}_1 + \boldsymbol{A}_{12}{}^i \boldsymbol{x}_2 \\ & \qquad\qquad \dot{\boldsymbol{x}}_2 = \boldsymbol{A}_{21}{}^i \boldsymbol{x}_1 + \boldsymbol{A}_{22}{}^i \boldsymbol{x}_2 + \boldsymbol{B}_2 \boldsymbol{u} \\ & \qquad \text{for } i = 1, \ldots\ldots m, \end{aligned} \tag{1}$$

where R^i represents the i^{th} fuzzy inference rule, m is the number of inference rules, $F_j \ (j = 1,\ldots n)$ are fuzzy sets, $\boldsymbol{x}_1 \in \Re^{n-p}$ and $\boldsymbol{x}_2 \in \Re^{p}$ are the system state variable vector, $\boldsymbol{u} \in \Re^{p}$ is the control input, $\boldsymbol{A}_{11}{}^i \in \Re^{(n-p)\times(n-p)}$, $\boldsymbol{A}_{12}{}^i \in \Re^{(n-p)\times p}$, $\boldsymbol{A}_{21}{}^i \in \Re^{p\times(n-p)}$, $\boldsymbol{A}_{22}{}^i \in \Re^{p\times p}$, and $\boldsymbol{B}_2 \in \Re^{p\times p}$ are the system matrices, $\boldsymbol{B}_2$ is non-singular, and $\boldsymbol{x} = (x_1, \ldots, x_n)^T$.

Denote μ^i as the normalized fuzzy membership function

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$$F^i = \prod_{j=1}^{n} F_j^i \;,\; \mu^i = \frac{F^i}{\sum_{i=1}^{m} F^i} \;,\; \text{and} \;\; \sum_{i=1}^{m} \mu^i = 1 \,. \qquad (2)$$

Using the fuzzy inference method with a center-average defuzzifier, product inference and singleton fuzzifier, the MIMO T-S fuzzy system (1) can be expressed by the following global model:

$$\begin{aligned} \dot{\boldsymbol{x}}_1 &= \boldsymbol{A}_{11}(\mu)\boldsymbol{x}_1 + \boldsymbol{A}_{12}(\mu)\boldsymbol{x}_2 \\ \dot{\boldsymbol{x}}_2 &= \boldsymbol{A}_{21}(\mu)\boldsymbol{x}_1 + \boldsymbol{A}_{22}(\mu)\boldsymbol{x}_2 + \boldsymbol{B}_2\boldsymbol{u} \end{aligned} \qquad (3)$$

where

$$\begin{aligned} &\boldsymbol{A}_{11}(\mu) = \sum_{i=1}^{m} \mu^i \boldsymbol{A}_{11}{}^i, \;\; \boldsymbol{A}_{12}(\mu) = \sum_{i=1}^{m} \mu^i \boldsymbol{A}_{12}{}^i, \\ &\boldsymbol{A}_{21}(\mu) = \sum_{i=1}^{m} \mu^i \boldsymbol{A}_{21}{}^i, \;\; \boldsymbol{A}_{22}(\mu) = \sum_{i=1}^{m} \mu^i \boldsymbol{A}_{22}{}^i, \\ &\mu = \left(\mu^1, \mu^2, ..., \mu^m\right). \end{aligned} \qquad (4)$$

For the further analysis, we let

$$\boldsymbol{A}^i = \begin{bmatrix} \boldsymbol{A}_{11}{}^i & \boldsymbol{A}_{12}{}^i \\ \boldsymbol{A}_{21}{}^i & \boldsymbol{A}_{22}{}^i \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 0 \\ \boldsymbol{B}_2 \end{bmatrix}, \qquad (5a)$$

$$\boldsymbol{A}(\mu) = \begin{bmatrix} \boldsymbol{A}_{11}(\mu) & \boldsymbol{A}_{12}(\mu) \\ \boldsymbol{A}_{21}(\mu) & \boldsymbol{A}_{22}(\mu) \end{bmatrix}. \qquad (5b)$$

Before we proceed, we have the following assumptions:

Assumption 1: Each linear subsystem in (5a) is controllable, i.e. the controllability matrices $\boldsymbol{M}^i = \left[\boldsymbol{B}, \boldsymbol{A}^i\boldsymbol{B}, \left(\boldsymbol{A}^i\right)^2 \boldsymbol{B}, ..., \left(\boldsymbol{A}^i\right)^{n-1} \boldsymbol{B}\right]$ for $i = 1,...,m$ have full rank.

Assumption 2: The global fuzzy model (5b) is controllable in the state space, i.e. the controllability matrix $\boldsymbol{M} = \left[\boldsymbol{B}, \boldsymbol{A}(\mu)\boldsymbol{B}, ..., \boldsymbol{A}^{n-1}(\mu)\boldsymbol{B}\right]$ has full rank in the state space [13].

III. EXTREME MATRICES

Following [12, 13], we decompose the state-space of the MIMO T-S fuzzy system (3) into *m* subspaces as follows:

$$\mathbb{S}_i = \left\{ x \mid \mu^i \geq \mu^l, l = 1,2,...,m,\; l \neq i \right\}, \;\text{for}\; i = 1,2,...,m \,. \qquad (6)$$

The characteristic function of $\mathbb{S}_i$ is defined by

$$\eta^i = \begin{cases} 1 (x \in \mathbb{S}_i), \\ 0 (x \notin \mathbb{S}_i). \end{cases} \qquad (7)$$

Let G be the set of membership functions satisfying (2). Then, the global model of the MIMO T-S fuzzy system can be expressed in each subspace as

$$\begin{aligned} \dot{\boldsymbol{x}}_1 &= [\boldsymbol{A}_{11}{}^i + \nabla\boldsymbol{A}_{11}{}^i(\mu)]\boldsymbol{x}_1 + [\boldsymbol{A}_{12}{}^i + \nabla\boldsymbol{A}_{12}{}^i(\mu)]\boldsymbol{x}_2 \\ \dot{\boldsymbol{x}}_2 &= [\boldsymbol{A}_{21}{}^i + \nabla\boldsymbol{A}_{21}{}^i(\mu)]\boldsymbol{x}_1 + [\boldsymbol{A}_{22}{}^i + \nabla\boldsymbol{A}_{22}{}^i(\mu)]\boldsymbol{x}_2 + \boldsymbol{B}_2\boldsymbol{u} \,, \end{aligned} \qquad (8)$$

where

$$\begin{aligned} &\nabla\boldsymbol{A}_{jk}{}^i(\mu) = \sum_{l=1}^{m_l} \bar{\mu}^l \nabla\boldsymbol{A}_{jk}{}^{il}, \;\; \nabla\boldsymbol{A}_{jk}{}^{il} = \boldsymbol{A}_{jk}{}^l - \boldsymbol{A}_{jk}{}^i, \\ &\bar{\mu}^l \in G, \quad l = 1,...,m, \quad \bar{\mu}^l \neq \mu^i, \\ &\bar{\mu}^l \neq 0, \quad \forall x \in \mathbb{S}_i, \\ &\text{for}\; i = 1,2,...,m,\; j = 1,2, \text{ and } k = 1,2. \end{aligned} \qquad (9)$$

Obviously, the interactions of the fuzzy subsystems are represented by $\nabla\boldsymbol{A}_{jk}{}^i(\mu)$. Here the i^{th} subsystem (8) is different from the fuzzy dynamical local model in (3), because it considers all the interactions among the local fuzzy models.

Using the characteristic function of $\mathbb{S}_i$, (8) can be denoted by

$$\begin{aligned} \dot{\boldsymbol{x}}_1{}^i &= [\boldsymbol{A}_{11}{}^i + \nabla\boldsymbol{A}_{11}{}^i(\mu)]\boldsymbol{x}_1{}^i + [\boldsymbol{A}_{12}{}^i + \nabla\boldsymbol{A}_{12}{}^i(\mu)]\boldsymbol{x}_2{}^i \\ \dot{\boldsymbol{x}}_2{}^i &= [\boldsymbol{A}_{21}{}^i + \nabla\boldsymbol{A}_{21}{}^i(\mu)]\boldsymbol{x}_1{}^i + [\boldsymbol{A}_{22}{}^i + \nabla\boldsymbol{A}_{22}{}^i(\mu)]\boldsymbol{x}_2{}^i + \boldsymbol{B}_2\boldsymbol{w}^i \,, \end{aligned} \qquad (10)$$

where $i = 1,2,...,m$, $\boldsymbol{x}^i = \eta^i\boldsymbol{x}$, and $\boldsymbol{w}^i = \eta^i\boldsymbol{u}$.

It can be seen that the i^{th} subsystem is time-varying. For the design of a TSM controller, we define the following upper bounds:

$$(\nabla\boldsymbol{A}_{jk}^i(\mu))^T \nabla\boldsymbol{A}_{jk}^i(\mu) \leq (\boldsymbol{E}_{jk}^i)^T \boldsymbol{E}_{jk}^i \,, \qquad (11)$$

for $i = 1,2,...,m$, $j = 1,2$, and $k = 1,2$. Here we call the matrices, $\boldsymbol{E}_{jk}^i$, the extreme matrices.

IV. TSM FOR MIMO T-S FUZZY SYSTEMS

In this section, we will develop a TSM controller for MIMO T-S fuzzy systems introduced in section 2. In order to obtain the terminal convergence of the state variables, we have the following definitions.

Definition 1: The TSM surface can be described by the following first order nonlinear differential equations [14]

$$s = \dot{x} + \beta |x|^{q/Q} sign(x) = 0 \;, \qquad (12)$$

where $x \in \Re$, α, $\beta > 0$, q and Q are odd integers and $Q > q$.

Definition 2: The equilibrium point $x = 0$ of the differential equation (12) is globally finite-time stable, i.e., for any given initial condition $x(0) = x_0$, the system state converges to $x = 0$ in finite time [14]

$$T = \frac{1}{\beta(1 - q/Q)} |x_0|^{1-Q} \,, \qquad (13)$$

and stays there forever.

Based on [10], the TSM surface for the MIMO T-S fuzzy system in the i^{th} subspace is defined as

$$\boldsymbol{s}^i = \boldsymbol{c}_1^i\boldsymbol{x}_1 + \boldsymbol{c}_2^i\boldsymbol{x}_2 + \boldsymbol{c}_3^i\boldsymbol{x}_1{}^{q/Q} = 0 \,, \qquad (14)$$

where $c_1^i \in \Re^{p\times(n-p)}$, $c_2^i \in \Re^{p\times p}$, and $c_3^i \in \Re^{p\times(n-p)}$ are the parameter matrices of the TSM surface.

Lemma 1: (Man and Yu [10]) If

$$2q > Q, \tag{15}$$

then the control input $\boldsymbol{w}^i$ is bounded, and hence $\boldsymbol{u}$ is bounded.

For the further analysis, we have the following assumptions.

Assumption 3: The system matrices $\left[(A_{12}{}^i)^T A_{12}{}^i\right]$ and $\left[A_{12}{}^i (A_{12}{}^i)^T\right]$ are non-singular for $i = 1,...,m$.

Assumption 4: The interactions of the fuzzy subsystems $\nabla A_{12}{}^i(\mu)$ in the i^{th} subspace are bounded in Euclidean norm with $\left\|\nabla A_{12}{}^i(\mu)((A_{12}^i)^T A_{12}^i)^{-1}(A_{12}^i)^T\right\| < 1$.

Assumption 5: $E_{12}{}^i$ are designed such that not only condition (11) is satisfied, but also it is bounded in Euclidean norm with $\left\|E_{12}{}^i((A_{12}^i)^T A_{12}^i)^{-1}(A_{12}^i)^T\right\| < 1$.

To obtain the sufficient condition of the existence of the TSM surface (14) for the MIMO T-S fuzzy system in the i^{th} subspace, we have the following theorem.

Theorem 1: Consider the MIMO T-S fuzzy system in (3). If the control input for the i^{th} subsystem is designed as:

$$\boldsymbol{w}^i = -(\boldsymbol{c}_2^i \boldsymbol{B}_2)^{-1}\left\{\boldsymbol{k}_1{}^i + sign(\boldsymbol{s}^i)\boldsymbol{k}_2{}^i + \beta^i \begin{bmatrix} s_1^{(q_s/Q_s)i} \\ \vdots \\ s_m^{(q_s/Q_s)i} \end{bmatrix}\right\}, \tag{16a}$$

where

$$\boldsymbol{k}_1{}^i = \boldsymbol{c}_1^i \boldsymbol{A}_{11}^i \boldsymbol{x}_1 + \boldsymbol{c}_1^i \boldsymbol{A}_{12}^i \boldsymbol{x}_2 + \boldsymbol{c}_2^i \boldsymbol{A}_{21}^i \boldsymbol{x}_1 + \boldsymbol{c}_2^i \boldsymbol{A}_{22}^i \boldsymbol{x}_2 + \boldsymbol{c}_3^i (q/Q) diag(x_l^{(q-Q)/Q})(\boldsymbol{A}_{11}^i \boldsymbol{x}_1 + \boldsymbol{A}_{12}^i \boldsymbol{x}_2), \tag{16b}$$

$$\boldsymbol{k}_2{}^i = \left\|\boldsymbol{c}_1^i \boldsymbol{E}_{11}^i \boldsymbol{x}_1\right\| + \left\|\boldsymbol{c}_1^i \boldsymbol{E}_{12}^i \boldsymbol{x}_2\right\| + \left\|\boldsymbol{c}_2^i \boldsymbol{E}_{21}^i \boldsymbol{x}_1\right\| + \left\|\boldsymbol{c}_2^i \boldsymbol{E}_{22}^i \boldsymbol{x}_2\right\| + \left\|\boldsymbol{c}_3^i (q/p) diag(x_l^{(q-Q)/Q})(\boldsymbol{E}_{11}^i \boldsymbol{x}_1 + \boldsymbol{E}_{12}^i \boldsymbol{x}_2)\right\|, \tag{16c}$$

$$diag(x_l^{(q-Q)/Q}) = \begin{bmatrix} x_1^{(q-Q)/Q} & & \\ & \ddots & \\ & & x_{n-p}^{(q-Q)/Q} \end{bmatrix}, \tag{16d}$$

then the terminal sliding variable vector $\boldsymbol{s}^i$ will reach the TSM surface $\boldsymbol{s}^i = 0$ in finite time

$$T_s \le \sum_i T_s^i, \tag{16e}$$

where

$$T_s^i = \frac{1}{\beta^i(1 - q_s/Q_s)}\left|\left(\boldsymbol{s}^i(0)\right)^T \boldsymbol{s}^i(0)\right|^{1-q_s/Q_s}. \tag{16f}$$

Proof: Defining a Lyapunov function

$$V^i = 1/2(\boldsymbol{s}^i)^T \boldsymbol{s}^i \tag{17}$$

and differentiating V^i with respect to time, we have

$$\begin{aligned}
\dot{V}^i &= (\boldsymbol{s}^i)^T \dot{\boldsymbol{s}}^i \\
&= (\boldsymbol{s}^i)^T\left[\boldsymbol{c}_1^i \dot{\boldsymbol{x}}_1 + \boldsymbol{c}_2^i \dot{\boldsymbol{x}}_2 + \boldsymbol{c}_3^i (q/Q) diag(x_l^{(q-Q)/Q})\dot{\boldsymbol{x}}_1\right] \\
&= (\boldsymbol{s}^i)^T\Big[\boldsymbol{c}_1^i[\boldsymbol{A}_{11}{}^i + \nabla\boldsymbol{A}_{11}{}^i(\mu)]\boldsymbol{x}_1{}^i + \boldsymbol{c}_1^i[\boldsymbol{A}_{12}{}^i + \nabla\boldsymbol{A}_{12}{}^i(\mu)]\boldsymbol{x}_2{}^i \\
&\quad + \boldsymbol{c}_2^i[\boldsymbol{A}_{21}{}^i + \nabla\boldsymbol{A}_{21}{}^i(\mu)]\boldsymbol{x}_1{}^i + \boldsymbol{c}_2^i[\boldsymbol{A}_{22}{}^i + \nabla\boldsymbol{A}_{22}{}^i(\mu)]\boldsymbol{x}_2{}^i \\
&\quad + \boldsymbol{c}_2^i \boldsymbol{B}_2 \boldsymbol{w}^i + \boldsymbol{c}_3^i (q/Q) diag(x_l^{(q-Q)/Q})[\boldsymbol{A}_{11}{}^i + \nabla\boldsymbol{A}_{11}{}^i(\mu)]\boldsymbol{x}_1{}^i \\
&\quad + \boldsymbol{c}_3^i (q/Q) diag(x_l^{(q-Q)/Q})[\boldsymbol{A}_{12}{}^i + \nabla\boldsymbol{A}_{12}{}^i(\mu)]\boldsymbol{x}_2{}^i\Big] \\
&= (\boldsymbol{s}^i)^T\Big[\boldsymbol{k}_1{}^i + \boldsymbol{c}_1^i \nabla\boldsymbol{A}_{11}{}^i(\mu)\boldsymbol{x}_1{}^i + \boldsymbol{c}_1^i \nabla\boldsymbol{A}_{12}{}^i(\mu)\boldsymbol{x}_2{}^i \\
&\quad + \boldsymbol{c}_2^i \nabla\boldsymbol{A}_{21}{}^i(\mu)\boldsymbol{x}_1{}^i + \boldsymbol{c}_2^i \nabla\boldsymbol{A}_{22}{}^i(\mu)\boldsymbol{x}_2{}^i + \boldsymbol{c}_2^i \boldsymbol{B}_2 \boldsymbol{w}^i \\
&\quad + \boldsymbol{c}_3^i (q/Q) diag(x_l^{(q-Q)/Q})\nabla\boldsymbol{A}_{11}{}^i(\mu)\boldsymbol{x}_1{}^i \\
&\quad + \boldsymbol{c}_3^i (q/Q) diag(x_l^{(q-Q)/Q})\nabla\boldsymbol{A}_{12}{}^i(\mu)\boldsymbol{x}_2{}^i\Big] \\
&\le \left\|\boldsymbol{s}^i\right\|\left\|\boldsymbol{c}_1^i \nabla\boldsymbol{A}_{11}{}^i(\mu)\boldsymbol{x}_1{}^i\right\| + \left\|\boldsymbol{s}^i\right\|\left\|\boldsymbol{c}_1^i \nabla\boldsymbol{A}_{12}{}^i(\mu)\boldsymbol{x}_2{}^i\right\| \\
&\quad + \left\|\boldsymbol{s}^i\right\|\left\|\boldsymbol{c}_2^i \nabla\boldsymbol{A}_{21}{}^i(\mu)\boldsymbol{x}_1{}^i\right\| + \left\|\boldsymbol{s}^i\right\|\left\|\boldsymbol{c}_2^i \nabla\boldsymbol{A}_{22}{}^i(\mu)\boldsymbol{x}_2{}^i\right\| - \left\|\boldsymbol{s}^i\right\|\boldsymbol{k}_2{}^i \\
&\quad + \left\|\boldsymbol{s}^i\right\|\Big\|\boldsymbol{c}_3^i (q/Q) diag(x_l^{(q-Q)/Q})\nabla\boldsymbol{A}_{11}{}^i(\mu)\boldsymbol{x}_1{}^i \\
&\quad + \boldsymbol{c}_3^i (q/Q) diag(x_l^{(q-Q)/Q})\nabla\boldsymbol{A}_{12}{}^i(\mu)\boldsymbol{x}_2{}^i\Big\| \\
&\quad - \beta^i(\boldsymbol{s}^i)^T\left[s_1^{(q_s/Q_s)i} \quad \cdots \quad s_m^{(q_s/Q_s)i}\right]^T \\
&\le -2\beta^i(V^i)^{(q_s+Q_s)/2Q_s}.
\end{aligned} \tag{18}$$

According to the finite time stability criteria (12), TSM surface (14) will be reached in the finite time (16e).

The design of the parameter matrices of the TSM variable in (14) and the convergence property of the MIMO T-S fuzzy system on the TSM surface are stated in the following theorem.

Theorem 2: Consider the MIMO T-S fuzzy system in (3) with the TSM controller in (16a) under Assumptions 3-5. If the TSM parameter matrices are designed such that

$$\boldsymbol{c}_1^i = (\boldsymbol{A}_{12}^i)^T(\boldsymbol{A}_{12}^i(\boldsymbol{A}_{12}^i)^T)^{-1}\left\|\boldsymbol{A}^i{}_{11} + \boldsymbol{E}^i{}_{11}\right\|\boldsymbol{I} \tag{19a}$$

$$\boldsymbol{c}_2^i = \left(1 - \left\|\boldsymbol{E}_{12}^i((\boldsymbol{A}_{12}^i)^T \boldsymbol{A}_{12}^i)^{-1}(\boldsymbol{A}_{12}^i)^T\right\|\right)\boldsymbol{I} \tag{19b}$$

$$\boldsymbol{c}_3^i = (\boldsymbol{A}_{12}^i)^T(\boldsymbol{A}_{12}^i(\boldsymbol{A}_{12}^i)^T)^{-1} diag(\beta_l^i), \tag{19c}$$

then the system states will converge to zero in finite time on the TSM surface.

Proof: On the TSM surface (14), the state variable vector $\boldsymbol{x}_2$ can be written as

$$\boldsymbol{x}_2 = -\frac{(\boldsymbol{A}_{12}^i)^T(\boldsymbol{A}_{12}^i(\boldsymbol{A}_{12}^i)^T)^{-1}}{1-\left\|\boldsymbol{E}_{12}^i((\boldsymbol{A}_{12}^i)^T\boldsymbol{A}_{12}^i)^{-1}(\boldsymbol{A}_{12}^i)^T\right\|}\Big[\left\|\boldsymbol{A}^i{}_{11}+\boldsymbol{E}^i{}_{11}\right\|\boldsymbol{I}\boldsymbol{x}_1 + diag(\beta_l^i)\boldsymbol{x}_1{}^{q/Q}\Big]. \tag{20}$$

It is seen that $\boldsymbol{x}_2$ is the linear combinations of $\boldsymbol{x}_1$. Therefore to prove that the system states can converge to zero in finite time on the TSM surface, we only need to show that $\boldsymbol{x}_1$ can reach the system origin in finite time.

Consider the Lyapunov function

$$V = 1/2\,\boldsymbol{x}_1{}^T\boldsymbol{x}_1. \tag{21}$$

Differentiating V with respect to time and using (10) and (20), we have

$$\begin{aligned}
\dot{V} &= \boldsymbol{x}_1{}^T\dot{\boldsymbol{x}}_1 \\
&= \boldsymbol{x}_1{}^T\Big[[\boldsymbol{A}_{11}{}^i+\nabla\boldsymbol{A}_{11}{}^i(\mu)]\boldsymbol{x}_1 \\
&\quad -[\boldsymbol{I}+\nabla\boldsymbol{A}_{12}{}^i(\mu)((\boldsymbol{A}_{12}^i)^T\boldsymbol{A}_{12}^i)^{-1}(\boldsymbol{A}_{12}^i)^T] \\
&\quad \times\frac{\boldsymbol{A}_{12}^i(\boldsymbol{A}_{12}^i)^T(\boldsymbol{A}_{12}^i(\boldsymbol{A}_{12}^i)^T)^{-1}}{\left\|\boldsymbol{I}-\boldsymbol{E}_{12}^i((\boldsymbol{A}_{12}^i)^T\boldsymbol{A}_{12}^i)^{-1}(\boldsymbol{A}_{12}^i)^T\right\|}\Big[\Big\|\boldsymbol{A}^i{}_{11} \\
&\quad +\boldsymbol{E}^i{}_{11}\Big\|\boldsymbol{I}\boldsymbol{x}_1+diag(\beta_l^i)\boldsymbol{x}_1{}^{q/Q}\Big]\Big] \\
&\le \left\|\boldsymbol{A}_{11}{}^i+\nabla\boldsymbol{A}_{11}{}^i(\mu)\right\|\left\|\boldsymbol{x}_1\right\|^2 \\
&\quad -\frac{1-\left\|\nabla\boldsymbol{A}_{12}{}^i(\mu)((\boldsymbol{A}_{12}^i)^T\boldsymbol{A}_{12}^i)^{-1}(\boldsymbol{A}_{12}^i)^T\right\|}{1-\left\|\boldsymbol{E}_{12}^i((\boldsymbol{A}_{12}^i)^T\boldsymbol{A}_{12}^i)^{-1}(\boldsymbol{A}_{12}^i)^T\right\|}\Big[\Big\|\boldsymbol{A}^i{}_{11} \\
&\quad +\boldsymbol{E}^i{}_{11}\Big\|\left\|\boldsymbol{x}_1\right\|^2+diag(\beta_l^i)\boldsymbol{x}_1{}^T\boldsymbol{x}_1{}^{q/Q}\Big] \\
&< -diag(\beta_l^i)\boldsymbol{x}_1{}^T\boldsymbol{x}_1{}^{q/Q} \\
&= -2\beta^i V^{(q+Q)/2Q}.
\end{aligned} \tag{22}$$

According to the finite time stability criteria (12), the system states $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ will reach the system origin in finite time.

V. SIMULATION EXAMPLE

To illustrate the finite time stabilization property of the proposed TSM controller, we consider the following MIMO T-S fuzzy system:

R^1: *IF* x_1 *is* F_1

THEN

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}1&0&1&0\\0&1&0&1\\1&0&0&0\\-1&-2&-3&-4\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} + \begin{bmatrix}0&0\\0&0\\0&1\\1&0\end{bmatrix}\begin{bmatrix}u_1\\u_2\end{bmatrix} \tag{23a}$$

R^2: *IF* x_1 *is* F_2

THEN

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}1&0&1&0\\0&1&0&1\\1&0&0&0\\3&4&5&6\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} + \begin{bmatrix}0&0\\0&0\\0&1\\1&0\end{bmatrix}\begin{bmatrix}u_1\\u_2\end{bmatrix}. \tag{23b}$$

We use the following membership functions, F_1 and F_2:

$$F_1 = \frac{1-1/(1+\exp(-14(x_1-\pi/8)))}{1+\exp(-14(x_1+\pi/8))}$$
$$F_2 = 1-F_1.$$

In this simulation, we choose

$$\boldsymbol{A}_{11}^1 = \boldsymbol{A}_{11}^2 = \begin{bmatrix}1&0\\0&1\end{bmatrix},\ \boldsymbol{A}_{12}^1 = \boldsymbol{A}_{12}^2 = \begin{bmatrix}1&0\\0&1\end{bmatrix},$$
$$\boldsymbol{A}_{21}^1 = \begin{bmatrix}1&0\\-1&-2\end{bmatrix},\ \boldsymbol{A}_{21}^2 = \begin{bmatrix}1&0\\3&4\end{bmatrix},$$
$$\boldsymbol{A}_{22}^1 = \begin{bmatrix}0&0\\-3&-4\end{bmatrix},\ \boldsymbol{A}_{22}^2 = \begin{bmatrix}0&0\\5&6\end{bmatrix},\ \boldsymbol{B}_2 = \begin{bmatrix}0&1\\1&0\end{bmatrix}.$$

The extreme matrices are selected as:

$$E_{11}^1 = E_{11}^2 = \begin{bmatrix}0.5&0\\0&0.5\end{bmatrix},\ E_{12}^1 = E_{12}^2 = \begin{bmatrix}0.5&0\\0&0.5\end{bmatrix},$$
$$E_{21}^1 = E_{21}^2 = \begin{bmatrix}1&0\\2&3\end{bmatrix},\ E_{22}^1 = E_{22}^2 = \begin{bmatrix}0&0.5\\4&5\end{bmatrix}.$$

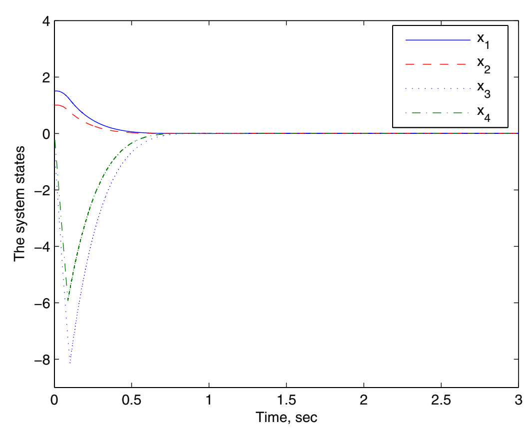

Figure 1. The system states

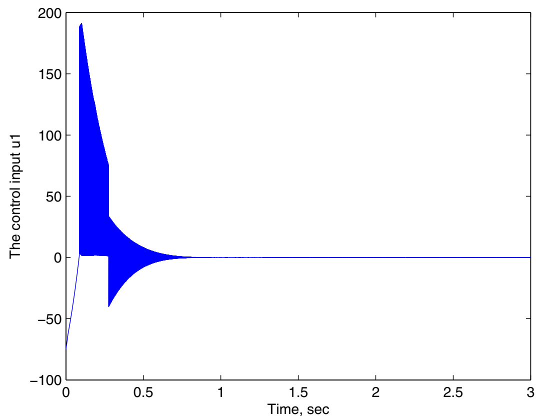

Figure 2. The control input u_1

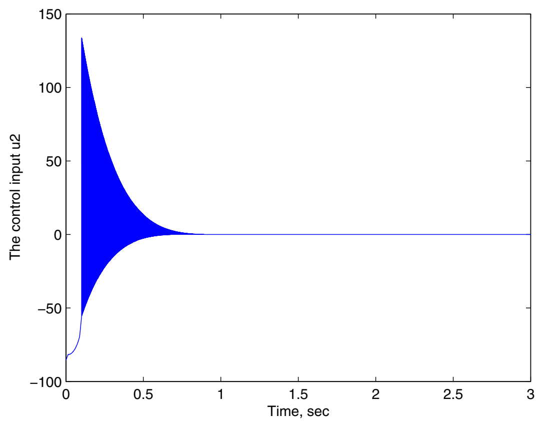

Figure 3. The control input u_2

Simulation results shown in Fig. 1 illustrate the effectiveness of the TSM controller (16a). Figs. 2 and 3 show the control inputs u_1 and u_2, respectively. It is seen that the good system performance has been achieved using the proposed control scheme. Finite time convergence property is guaranteed.

VI. CONCLUSIONS

A terminal sliding mode control scheme has been developed in this paper for MIMO T-S fuzzy systems. The extreme matrices have been used to design not only the TSM controller, but also the TSM surface. It has been shown that the TSM controller is able to drive system states to reach the system origin in finite time. A simulation example has been given in support of the proposed control scheme.

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